Haar-$\mathcal{I}$ sets

Jarosław Swaczyna

Łódź University of Technology

joint work with Szymon Głąb, Eliza Jabłońska and Taras Banakh (in progress)

I start with recalling ”Summer Symposium in Real Analysis” conference

During which I probably found the main point which makes me interested in things in math

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I started with fractals (so contractions) and then turned to ideals

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as you can see, I am not an expert in being small, but I would strongly appreciate getting some knowledge in area :-)

Background

$(G, +)$ - abelian Polish group

We look for measure-like notion of smallness for G.

G is locally compact iff there exists regular invariant Borel measure (so-called Haar measure) which is unique up to multiplying by constant.

Hence in locally compact Polish groups we obtain natural notion of null sets.

If considered group is not locally compact there is no distinguished measure on it, however analoguos notion (so-called Haar null sets) were introduced by Christensen in 1972. His notion was rediscovered by Hunt, Sauer and Yorke in 1992 and since that time it was deeply investigated.

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Definitions

We say that set $A \subset G$ is *Haar-null*, or $A \in \mathcal{HN}(G)$, if there exists Borel hull $B \supset A$ and a Borel probalistic measure μ on G such that for any $g \in G$ we have $\mu(B + g) = 0$. We say that μ witnesses the fact that A is Haar-null. In original Christensen's paper hull B was supposed only to be universally measureable.

Theorem (Christensen)

Haar-null sets forms a proper σ-ideal. If G is locally compact, they coincide with Haar-measure null sets.

Observation

For any $A \in \mathcal{HN}$:

- measure which witnesses this fact is continuous,
- A has empty interior,
- there exist compactly (or even Cantorly) supported measure which witnesses this fact.

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Definitions

Darji, 2013

We say that set $A \subset G$ is *Haar-meager*, or $A \in \mathcal{HM}(G)$, if there exists Borel hull $B \supset A$ and a continuous function $h : 2^\omega \to G$ such that for any $g \in G$ we have $h^{-1}(B+g) \in \mathcal{M}_{2^\omega}$. We say that h witnesses the fact that A is Haar-meager.

Theorem (Darji)

Haar-meager sets forms a σ-subideal of meager sets. Those notions coincides iff G is locally compact.

We say that set $A \subset G$ is *Darji-Haar-null*, or $A \in \mathcal{DHN}(G)$, if there exists Borel hull $B \supset A$ and a continuous function $h : 2^\omega \to G$ such that for any $x \in G$ we have $h^{-1}(B+g) \in \mathcal{N}_{2^\omega}$. We say that h witnesses the fact that A is Darji-Haar-meager.

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Haar-small sets

Theorem

$\mathcal{DHN} = \mathcal{HN}$

Now it seems reasonable to consider following definition:

Definition

Let $\mathcal{I}$ be (σ-)ideal on 2^ω. We will say that $A \subset G$ is Haar-$\mathcal{I}$-small ($A \in \mathcal{HI}$) if there exists Borel? hull $B \supset A$ and continuous $f : 2^\omega \to G$ such that $f^{-1}(B + g) \in \mathcal{I}$ for all $g \in G$.

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Basic facts

Theorem

Let ν be a continuous, fully supported Borel probabilistic measure on 2^ω. There exists order-preserving homeomorphism $f : 2^\omega \to 2^\omega$ for which $f(\mathcal{N}_\nu) = \mathcal{N}_\lambda$.

Corollary

If μ is σ-finite Borel measure on 2^ω, then $\mathcal{HN}_\mu = \mathcal{HN}$ or $\mathcal{HN} = \{\emptyset\}$.

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Problem

When $\mathcal{HI}$ is an ideal?

Definition

We say that $\mathcal{I}$ has *Fubini property*, if there exists such a homeomorphism $h : 2^\omega \to (2^\omega)^\omega$ that

$$\forall_{A \in Borel(2^\omega)} \left[\left(\exists_{j \in \omega} \forall_{(t_i)_{i \neq j} \in (2^\omega)^\omega} A_{(t_i)_{i \neq j}} \in \mathcal{I} \right) \Rightarrow h^{-1}(A) \in \mathcal{I} \right]$$

where $A_{(t_i)_{i \neq j}} = \{t \in 2^\omega : (t_0, \ldots, t_{j-1}, t, t_{j+1}, \ldots) \in A\}$.

Theorem

If $\mathcal{I}$ is a σ-ideal which has a Fubini property then $\mathcal{HI}$ is a σ-ideal.

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It is probably still far from characterization...

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Something easier

Is $\mathcal{HF}in$ an ideal?

We don't know.

Definition

If $A \in \mathcal{HI}$ and there exists an injection which witnesses this fact, we say A is *injectively Haar-$\mathcal{I}$* and write $A \in \mathcal{EHI}$. Clearly $\mathcal{EHI} \subset \mathcal{HI}$ for any $\mathcal{I}$ and G.

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Basic observations

- If $Fin \not\subset \mathcal{I}$, then $\mathcal{HI} = \emptyset$,
- $\mathcal{EH}Fin = \mathcal{H}Fin$,
- If analytic $Count \not\ni A \in \mathcal{H}Fin$ and f witnesses this fact, then $rng(f) \in \mathcal{H}Fin$,
- $\mathcal{I} \subset \mathcal{J} \implies \mathcal{HI} \subset \mathcal{HJ}$,
- If $A \in \mathcal{HI}$ and $f : G \to G$ is homeomorphism which preserves group action, then $f(A) \in \mathcal{HI}$. In particular, if G is a Banach space ideal $\mathcal{HI}$ is invariant for multiplying by a scalar.

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$\mathcal{HF}in$ - Example

For $G = \mathcal{I}R$ there exists a homeomorph C of the Cantor set with $C \in \mathcal{EH}Fin$.

$C := \{\sum_{n\in\omega} \frac{\epsilon_n}{7^n} : \forall_{n\in\omega}\epsilon_n \in \{1,2\}\}$, $D := \{\sum_{n\in\omega} \frac{\epsilon_n}{7^n} : \forall_{n\in\omega}\epsilon_n \in \{3,5\}\}$. $dim(C) = dim(D) = ln(2)/ln(7)$. Using similar mathod we may construct elements of $\mathcal{HF}in$ with Hausdorff dimension equal to $\ln(m-1)/ln(2m+1)$ for any $m \in \omega$. Mattila gave example of such a sets with Hausdorff dimension 1.

Observation

Assume that Polish group G can be decompose to form $G = \mathcal{I}R \times H$. Then there exists a homeomorph $A \subset \mathbb{R}$ of the Cantor set for which $dim(A) = 1$ and $A \times H \in \mathcal{EH}Fin$. In particular for each $n \in \omega$ there exists $A \subset \mathbb{R}^n$, $A \in \mathcal{EH}Fin$ with $dim(A) = n$.

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$C := \{\sum_{n\in\omega} \frac{\epsilon_n}{7^n} : \forall_{n\in\omega}\epsilon_n \in \{1,2\}\}$, $D := \{\sum_{n\in\omega} \frac{\epsilon_n}{7^n} : \forall_{n\in\omega}\epsilon_n \in \{3,5\}\}$. $dim(C) = dim(D) = ln(2)/ln(7)$. Using similar mathod we may construct elements of $\mathcal{HF}in$ with Hausdorff dimension equal to $\ln(m-1)/ln(2m+1)$ for any $m \in \omega$. Mattila gave example of such a sets with Hausdorff dimension 1.

Observation

Assume that Polish group G can be decompose to form $G = \mathcal{I}R \times H$. Then there exists a homeomorph $A \subset \mathbb{R}$ of the Cantor set for which $dim(A) = 1$ and $A \times H \in \mathcal{EHF}in$. In particular for each $n \in \omega$ there exists $A \subset \mathbb{R}^n$, $A \in \mathcal{EHF}in$ with $dim(A) = n$.

Haar-small sets

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Proof

Theorem

Let $f : G \to H$ be a Baire-measureable homomorphism between Polish groups with $rng(f) \notin \mathcal{M}_H$. If a set A is naively (injectively) Haar-$\mathcal{I}$ in H, then its preimage $f^{-1}(A)$ is naively (injectively) Haar-$\mathcal{I}$ set in G.

Ingredient1 - Lemma

For any $\mathcal{HI}$ set A and $\varepsilon > 0$ there exists witness f with $\mathrm{diam}(\mathrm{rng}(f))/\varepsilon$.

Ingredient2 - Michael SELECTION Theorem

If X is and zero-dimensional space, and if Y is a complete metric space, then every lower semi-continuous function φ from X to the non-empty, closed subsets of Y admits a continuous selection, where by lower semi-continuity we mean that set $\{x \in X : \varphi(x) \cap U \neq \emptyset\}$ is open for each open $U \subset Y$.

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Baire-measureable homomorphism between Polish groups with

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Why I am proving it here?

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Baire-measureable homomorphism between Polish groups with $rng(f) \notin \mathcal{M}_H$ are continuous and open.

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Take any $A \in \mathcal{HI}(\mathcal{EHI})$ in H and fix $w : 2^\omega \to H$ which witnesses this fact. Set $\varphi(x) := f^{-1}(w(x))$ for $x \in 2^\omega$. By openess of f there exists some open ball $U_0 \subset \mathrm{rng}(f)$ centered in some h_0 and by Ingredient1 we may assume that $\mathrm{diam}(\mathrm{rng}(w)) < \mathrm{diam}(U_0)/2$. By selecting any $h \in \mathrm{rng}(w)$ and considering $w' := w - h + h_0$ we may assume that $\mathrm{rng}(w) \subset U_0$. Thanks to this assumption we get that $\varphi(x) \neq \emptyset$ for each $x \in 2^\omega$, while by continuity of f $\varphi(x)$ is closed. In order to apply Ingredient2 we have to check if φ is lower semi-continuous, so fix open $U \subset G$. Since f is open mapping set $U_1 := f(U)$ is open, hence also $U_2 := w^{-1}(U_1)$ is open. Now observe that

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Hence by Ingredient2 we obtain continuous $s : 2^\omega \to G$ with $s(\cdot) \in f^{-1}(w(\cdot))$. Note that if w is injective then so is s.

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Dziękuję za uwagę! תודה לך על תשומת הלב
Thank you for your attention!
Děkuji za pozornost! Gracias por su atención!
Obrigado pela sua atenção!
Спасибо за внимание!
Ďakujem za tvoju pozornos'! شکریہ کا توجہ آپکی
Grazie per l'attenzione! Merci de votre attention !
Gratias pro vobis animus attentus!
Danke für Ihre Aufmerksamkeit!
Σας ευχαριστώ για την προσοχή σας!
İlginiz için teşekkür ederim! 感⊠您的关注!
ご清聴ありがとうございました！